

Hawking radiation from Elko particles tunnelling across black strings horizon

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We apply the tunnelling method for the emission and absorption of Elko particles in the event horizon of a black string solution. We show that Elko particles are emitted at the expected Hawking temperature from black strings, but with a quite different signature with respect to the Dirac particles. We employ the Hamilton-Jacobi technique to black hole tunnelling, by applying the WKB approximation to the coupled system of Dirac-like equations governing the Elko particle dynamics. As a typical signature, different Elko particles are shown to produce the same standard Hawking temperature for black strings. However we prove that they present the same probability irrespective of outgoing or ingoing the black hole horizon. It provides a typical signature for mass dimension one fermions, that is different from the mass dimension three halves fermions inherent to Dirac particles, as different Dirac spinor fields have distinct inward and outward probability of tunnelling.

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Black hole tunnelling procedures have been placed as prominent methods of calculating the temperature of black holes [1–14]. The tunnelling method imparts a dynamical model describing the black hole radiation, and has been applied to a plenty of black holes, both for the tunnelling of Dirac particles [12–18] and scalar particles as well [1, 2]. The first black hole tunnelling method [3] succeeded from the seminal result in [1, 2]. An alternative technique to study the tunnelling in black hole is the Hamilton-Jacobi one [4] by settling on a suitable ansatz for the action. This method was further extended, by applying the WKB approximation to the Dirac equation [14–18]. The black hole tunnelling method has strong points with respect to other routines of calculating temperature, and can be successfully further applied to other black holes [8, 9, 11, 12, 15, 19]. The tunnelling method provides a natural framework to study the black hole radiation, where a particle trails a path from the inside of the black hole to the outside, which is a banned possibility from the classical point of view. By energy conservation, the black hole radius constricts as a function of the energy of the outgoing particle, hence the particle provides its own tunnelling barrier [15, 16].

A quantum WKB approach was used to compute the corrections to the Hawking temperature and Bekenstein-Hawking entropy for the Schwarzschild black hole, modifying the Schwarzschild metric which takes into account effects of quantum corrections [20–23]. Furthermore, the black hole area was shown to have a lower bound [24] in tunnelling formalism. The chirality condition was likewise introduced to unify the anomaly and

the tunnelling formalisms for deriving the Hawking effect [25], and the Hawking radiation from the black hole, both in Hořava-Lifshitz and Einstein-Gauss-Bonnet gravities, was discussed in [26, 27]. Important achievements have been also accomplished in, e. g., [28] in a non-commutative framework.

The tunnelling method has been employed to provide Hawking radiation due to photon and gravitino tunnelling [29]. Moreover, this method was extended to model the emission of spin-1/2 fermions, and the Hawking radiation was deeply analyzed in [30] as tunnelling of Dirac particle throughout an event horizon, where quantum corrections in the single particle action are proportional to the usual semiclassical contribution and the modifications to the Hawking temperature and Bekenstein-Hawking entropy were derived for the Schwarzschild black hole. When fermions of spin-1/2 are taken into account, due to the fact that there are particles with the spin in any direction, the effect of the spin of each type of fermion countervails, and thus the lowest WKB order provides that the rotation of the black hole is unmodified. The authors in [7] argued that the probability of emission of a particle approaches zero when its energy becomes of the order of the mass of the emitting black hole. Consequently, the event horizon decreases [7, 11, 15], and the usual approximations used in the literature [1–18] remain to be adopted here.

Elko (dark) spinor fields (dual-helicity eigenspinors of the charge conjugation operator [31]) are spin-1/2 fermions of mass dimension one, with novel features that make them capable to incorporate both the Very Special Relativity (VSR) paradigm [32] and the dark matter description as well [31–35]. Moreover, an Elko spinor mass generation mechanism has been introduced in [36], by a natural coupling to the kink solution of a $\lambda\phi^4$ field theory. It provides exotic couplings among scalar field to

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pological solutions and Elko spinor fields [36, 37]. Some attempts to detect Elko at the LHC have been proposed [38–40], as well as important applications to cosmology have been widely investigated [41–46]. Not merely in quantum field theory, and supersymmetry [47], but additionally the Einstein-Hilbert, the Einstein-Palatini, and the Holst actions were shown to be derived from the quadratic spinor Lagrangian, when Elko spinor fields are considered [48, 49].

The tunnelling method is used in this paper to model Elko particles emission and absorption from black strings. We show that Elko particles are emitted at the expected Hawking temperature from black holes and black strings, providing further evidence for the universality of black hole radiation [11, 15, 16], however with a specific signature that is different from Dirac particles. In fact, we shall prove that Elko particles behave contrastively from Dirac particles, that present different inward and outward probability of tunnelling — depending on a relationship between the spinor components [17, 18]. In fact, we shall show that the four distinct Elko particles, being eigenspinors of the charge conjugation operator with dual helicity, manifest the property of presenting the same equations for tunnelling, and consequently the same inward and outward probability of tunnelling. Moreover, the standard Hawking temperature for black strings is obtained in this context. The results presented in this paper for Elko particles differ from Dirac particles, as naturally Elko particles are fields presenting mass dimension one [31, 33–35, 42].

String theory has solutions describing extra-dimensional extended objects surrounded by event horizons, namely black strings. These solutions can have unusual causal structure, and provide some insight into the properties of singularities in string theory. Black strings have been studied in the context of supergravity theories, topological defects and low energy string theories [50–52] and from pure gravitational context in [53, 54], as well as in some realistic generalizations [55–57].

Einstein equations has solutions [58, 59]

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2d\theta^2 + \alpha^2r^2dz^2, \quad (1)$$

where $\Lambda_{(4)} = -3\alpha^2$ denotes the cosmological constant, M is associated to the ADM mass density of the black string, and $f(r) = (\alpha^2r^2 - \frac{M}{r})$. The event horizon of the black hole is provided forthwith by:

$$\tilde{r} = \left(\frac{M}{\alpha^2}\right)^{1/3}.$$

Also, this solution was discussed in [63] in the context of Einstein-Maxwell gravity.

In order to analyze the tunnelling of fermions throughout the black string horizon, we depart from the usual mass dimension $3/2$ fermions, and shall investigate

the role that Elko particles play in this context. To accomplish it, the basic features of Elko particles are briefly revisited [31, 33, 61]. Elko spinor fields $\lambda(p^\mu)$ are eigenspinors of the charge conjugation operator C , namely, $C\lambda(p^\mu) = \pm\lambda(p^\mu)$ (here the momentum space is used just to fix the notation). The Weyl representation of γ^μ is used hereupon. The plus [minus] sign regards self-conjugate, [anti self-conjugate] spinor fields, denoted by $\lambda^S(p^\mu)$ [$\lambda^A(p^\mu)$]. Explicitly, once the rest spinors $\lambda(k^\mu)$ are obtained, for an arbitrary p^μ it yields

$$\lambda(p^\mu) = e^{i\kappa \cdot \varphi} \lambda(k^\mu), \quad (2)$$

where $k^\mu = \left(m, \lim_{p \rightarrow 0} \frac{\mathbf{p}}{p}\right)$, for $p = |\mathbf{p}|$. The boost operator in (2) is provided by [61]

$$e^{i\kappa \cdot \varphi} = \sqrt{\frac{E+m}{2m}} \text{diag} \left(\mathbb{I} + \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m}, \mathbb{I} - \frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{E+m} \right).$$

The $\phi(k^\mu)$ are defined to be eigenspinors of the helicity operator $\boldsymbol{\sigma} \cdot \hat{\mathbf{p}}$:

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{p}} \phi^\pm(k^\mu) = \pm \phi^\pm(k^\mu)$$

where $\hat{\mathbf{p}} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta)$, and the phases are employed [31, 33, 61] such that

$$\phi^+(k^\mu) = \sqrt{m} \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) e^{-i\phi/2} \\ \sin\left(\frac{\theta}{2}\right) e^{+i\phi/2} \end{pmatrix}, \quad (3)$$

$$\phi^-(k^\mu) = \sqrt{m} \begin{pmatrix} -\sin\left(\frac{\theta}{2}\right) e^{-i\phi/2} \\ \cos\left(\frac{\theta}{2}\right) e^{+i\phi/2} \end{pmatrix}. \quad (4)$$

Elko spinor fields $\lambda(k^\mu)$ are defined by

$$\lambda_\pm^S(k^\mu) = \begin{pmatrix} i\Theta[\phi^\pm(k^\mu)]^* \\ \phi^\pm(k^\mu) \end{pmatrix} \quad (5)$$

$$\lambda_\pm^A(k^\mu) = \pm \begin{pmatrix} -i\Theta[\phi^\mp(k^\mu)]^* \\ \phi^\mp(k^\mu) \end{pmatrix} \quad (6)$$

where the Θ denotes the Wigner time reversal operator for spin one half. Hereupon the notation $\phi^\pm(k^\mu) = \phi^\pm$ shall be used for the sake of simplicity. The expression

$$\boldsymbol{\sigma} \cdot \hat{\mathbf{p}} [\Theta(\phi^\pm)^*] = \mp [\Theta(\phi^\pm)^*]$$

evinces the helicity of $\Theta[\phi(k^\mu)]^*$ to be opposite to that of $\phi(k^\mu)$, and therefore

$$\lambda_\pm^S(p^\mu) = \sqrt{\frac{E+m}{2m}} \left(1 \mp \frac{p}{E+m} \right) \lambda_\pm^S \quad (7)$$

$$\lambda_\pm^A(p^\mu) = \sqrt{\frac{E+m}{2m}} \left(1 \pm \frac{p}{E+m} \right) \lambda_\pm^A \quad (8)$$

are the expansion coefficients of a mass dimension one quantum field. In fact, the Dirac operator $(\gamma_\mu p^\mu \pm m\mathbb{I}_4)$ does not annihilate the $\lambda(p^\mu)$ and the following results

hold [31, 33, 61]:

$$\gamma_\mu p^\mu \lambda_+^S(p^\mu) = im\lambda_-^S(p^\mu) \quad (9)$$

$$\gamma_\mu p^\mu \lambda_-^S(p^\mu) = -im\lambda_+^S(p^\mu) \quad (10)$$

$$\gamma_\mu p^\mu \lambda_-^A(p^\mu) = im\lambda_+^A(p^\mu) \quad (11)$$

$$\gamma_\mu p^\mu \lambda_+^A(p^\mu) = -im\lambda_-^A(p^\mu). \quad (12)$$

Nevertheless, it still implies annihilation of Elko by the Klein-Gordon operator.

Hawking radiation from black holes comprises different types of charged and uncharged particles. We investigate tunnelling of Elko particles from the event horizon of a black string solution via tunnelling formalism. By taking

$$\nabla_\mu = \partial_\mu + \omega_\mu, \quad \omega_\mu = \frac{1}{2}i\Gamma_\mu^{\alpha\beta}\sigma_{\alpha\beta},$$

where $\sigma_{\alpha\beta} = \frac{1}{4}i[\gamma^\alpha, \gamma^\beta]$ is the spin density tensor and the γ^μ are the usual gamma matrices satisfying the Clifford relation for Minkowski spacetime, the matrices

$$\gamma^t = \frac{1}{\sqrt{f}}\gamma^0, \quad \gamma^r = \sqrt{f}\gamma^3, \quad \gamma^\theta = \frac{1}{r}\gamma^1, \quad \gamma^z = \frac{1}{\alpha r}\gamma^2, \quad (13)$$

are chosen as usually [17], where $f = f(r)$. In order to find the solution of Eqs. (9)-(12) in the background of the black string, we employ the standard form for the Elko particle, through the similar notation $\phi^\pm = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$, where α and β defined in Eq. (3):

$$\lambda_+^S(t, r, \theta, z) = \begin{pmatrix} -i\beta^* \\ i\alpha^* \\ \alpha \\ \beta \end{pmatrix} \exp\left(\frac{i}{\hbar}\tilde{I}\right), \quad (14)$$

$$\lambda_-^S(t, r, \theta, z) = \begin{pmatrix} -i\alpha \\ -i\beta \\ -\beta^* \\ \alpha^* \end{pmatrix} \exp\left(\frac{i}{\hbar}\tilde{I}\right), \quad (15)$$

$$\lambda_+^A(t, r, \theta, z) = \begin{pmatrix} i\alpha \\ i\beta \\ -\beta^* \\ \alpha^* \end{pmatrix} \exp\left(\frac{i}{\hbar}\tilde{I}\right), \quad (16)$$

$$\lambda_-^A(t, r, \theta, z) = \begin{pmatrix} -i\beta^* \\ i\alpha^* \\ -\alpha \\ -\beta \end{pmatrix} \exp\left(\frac{i}{\hbar}\tilde{I}\right). \quad (17)$$

Here $\tilde{I} = \tilde{I}(t, r, \theta, z)$ represents the classical action. We use the above forms for the Elko particles in each one of the Eqs. (9)-(12), and solve this coupled system of equations. Thus, by applying the WKB approximation, where $\frac{i}{\hbar}\tilde{I} = \frac{i}{\hbar}I + I_0 + \mathcal{O}(\hbar)$, and considering terms solely up to the leading order in \hbar , by denoting $I_r = \partial I/\partial r$, $I_t = \partial I/\partial t$, $I_\theta = \partial I/\partial \theta$, and $I_z = \partial I/\partial z$, this procedure yields:

$$\frac{i\alpha^* I_t}{\sqrt{f}} + \beta\sqrt{f} I_r = m\beta^* + \left(\frac{i}{\alpha z}I_z - \frac{1}{r}I_\theta\right)\alpha^*, \quad (18)$$

$$\frac{i\beta I_t}{\sqrt{f}} - \alpha^*\sqrt{f} I_r = m\alpha^* - \left(\frac{i}{\alpha z}I_z + \frac{1}{r}I_\theta\right)\beta^*. \quad (19)$$

We can employ the usual ansatz in refs. [15–18]:

$$I(t, r, \theta, z) = -Et + W(r) + l\theta + Jz, \quad (20)$$

where E is the energy of the emitted particles and W is the part of the action \tilde{I} that contributes to the tunnelling probability. Using this ansatz in Eqs. (18, 19) [15–18], the terms in (18, 19) encompassing J and l are dismissed. The same solution for J is obtained for both the outgoing and incoming cases.

As it is comprehensively exposed in [15–18], near the black string horizon massive particles behave like massless particles. Phenomenologically, considering the well established Elko production by Higgs interactions [38–40], we proceed as refs. [15–18] and consider the parameter $m \approx 0$, without loss of generality, as near the horizon massive particles behave as massless ones. Thus, the function $W(r)$ can be computed merely from Eqs.(19) and (20) as:

$$\begin{aligned} -i\alpha^*E + \beta f(r)W'(r) &= 0, \\ -i\beta E + \alpha^* f(r)W'(r) &= 0. \end{aligned} \quad (21)$$

In this case for

$$\alpha = i\beta^* \quad (22)$$

we have

$$W'_+(r) = E/f(r), \quad (23)$$

whilst for the choice

$$\alpha = -i\beta^* \quad (24)$$

we get the opposite sign

$$W'_-(r) = -E/f(r). \quad (25)$$

W_+ [W_-] corresponds to outward [inward] solutions (see refs. [15–18]). Eqs. (23) and (25) imply that

$$W_\pm(r) = \pm \int (E/f(r))dr, \quad (26)$$

which has a simple pole at $r = \tilde{r}$. By integrating around the pole, it yields

$$W_\pm(r) = \frac{\pm\pi iE}{2\alpha^2\tilde{r} + \frac{M}{\tilde{r}^2}}. \quad (27)$$

The probabilities of crossing the horizon in each direction can be given by [4]

$$P_\pm \propto e^{-\frac{2}{\hbar}\text{Im } W_\pm(r)}, \quad (28)$$

where P_+ [P_-] denotes the probability of emission [absorption] by the horizon. While computing the imaginary part of the action, we note that it is the same for both the incoming and outgoing solutions. Eqs.(28) show now that

the probability of particles tunnelling from the inside to the outside of the event horizon is specified by

$$\Gamma \propto \frac{P_+}{P_-} = e^{-\frac{4}{\hbar} \text{Im} W_+(r)}, \quad (29)$$

where in the last equality we employed Eq.(27), implying that

$$\Gamma = \exp \left(-\frac{4\pi E}{2\alpha^2 \tilde{r} + \frac{M}{\tilde{r}^2}} \right). \quad (30)$$

As the tunnelling probability is given by $\Gamma = \exp(-\beta E)$, where $\beta = T_H^{-1}$, it yields the Hawking temperature formula

$$T_H = \frac{1}{4\pi} \left(2\alpha^2 \tilde{r} + \frac{M}{\tilde{r}^2} \right), \quad (31)$$

which is the usual Hawking temperature for black strings [62]. Massive particles behave like massless ones and since the extra contributions vanish at the horizon, the result of integrating around the pole for W_{\pm} in the massive case is the same as the massless case and the Hawking temperature is recovered. Moreover, as in the Dirac tunnelling, for both the massive and massless the Hawking temperature is obtained, implying that the Elko particles $\lambda_+^S, \lambda_-^A, \lambda_-^S, \lambda_+^A$ defined in Eqs.(5)-(8) — with explicit components in (14)-(17) — are emitted at the same rate. It endows Elko particles with a different signature with respect to the Dirac particles (see, e. g., refs. [15–18]), which we shall emphasize below.

In the tunnelling formalism the probability of particles crossing the black hole horizon on both sides are calculated using complex path integrals. Solving Elko coupled equations (9)-(12) in the background of black strings and by applying the WKB approximation, we have provided the tunnelling probability of Elko particles and the Hawking temperature associated to it.

Moreover, the tunnelling of Elko particles has a different feature when compared to Dirac particles. The method developed in [15, 16] for Dirac particles was further used in [17] in the context of black strings for the very special case where the spinor field is given by

$$\Psi_{\uparrow}(t, r, \theta, z) = \begin{pmatrix} A(t, r, \theta, z) \\ 0 \\ B(t, r, \theta, z) \\ 0 \end{pmatrix} \exp \left(\frac{i}{\hbar} \tilde{I} \right), \quad (32)$$

where the author shows that there is a constraint between A and B , similarly to (22) and (24). The inward and outward probability of tunnelling depends on the relation between A and B . For each constraint, Dirac particles present just one behavior: either ingoing or outgoing particles. Notwithstanding, Elko particles are eigenspinors of the charge conjugation operator, and all the eigenspinor fields ($\lambda_+^S, \lambda_-^A, \lambda_-^S, \lambda_+^A$) present the same probability either outgoing or ingoing for tunnelling. Notice that Elko spinor field λ_+^S in (14) differs from λ_-^A in (17) just by the sign in the left-handed component, whereas the Elko spinor field λ_-^S in (15) is different of λ_+^A in (16) by the sign in the right-handed component, although they are quite different quantum fields [61]. Moreover, all the four Elko particles present the same inward and outward probability of tunnelling and the standard Hawking temperature for black strings is obtained.

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